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Error Considerations in the Design of Microwave Transistor Amplifiers

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***Abstract*—In the design of microwave transistor amplifiers it is frequently of value to consider an idealization where the actual device is replaced by one with the reverse transfer parameter S_{12} set to zero, while other S-parameters remain unchanged. In this communication bounds are derived on errors which may result from use of such an idealization.**

I. Introduction

MANY DESIGN procedures for bipolar and field-effect microwave transistor amplifiers rely on the assumption that the reverse transfer parameter S_{12} may be set to zero while the remaining S-parameters are unchanged, e.g., [1], [2]. In this paper bounds are derived for the error which may be introduced as a result of this idealization. Conditions for the case of "zero error" between the gains from the "ideal" and "true" devices are also derived. Results are presented using S-parameter data for some representative microwave transistors, and amongst these examples the case of a potentially unstable device is included.

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II. Gain Relations

A. Gain Equalization

For the case of the amplifier layout of Fig. 1 the transducer power gain is [3]

$$G_T = \frac{|S_{21}|^2(1-|S_L|^2)(1-|S_g|^2)}{|1-S_gS_{11}-S_LS_{22}+S_LS_g\triangle|^2}, \qquad \triangle = S_{11}S_{22}-S_{12}S_{21} \tag{1}$$

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and with $S_{12}=0$

$$G_T=G_u=\frac{|S_{21}|^2(1-|S_L|^2)(1-|S_g|^2)}{|1-S_gS_{11}|^2|1-S_LS_{22}|^2}=\underbrace{\frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}}_{A}\underbrace{\frac{(1-|S_{11}|^2)(1-|S_g|^2)}{|1-S_gS_{11}|^2}}_{B}\underbrace{\frac{(1-|S_{22}|^2)(1-|S_L|^2)}{|1-S_LS_{22}|^2}}_{C} \quad (2)$$

with term A being the maximum unilateral gain $G_{u\max}$ and terms B and C representing renormalized transducer gains of input and output networks, respectively. $G_{u\max}$ is defined as the transducer gain of the idealized device ($S_{12}=0$) terminated at input and output ports by S_{11}^* and S_{22}^*, respectively.

It is usually required that the gain of the amplifier be flat over the band of interest. The device has an inherent gain–frequency rolloff where $G_{u\max}$ (maximum value of G_u) falls off at a rate of approximately 6 dB/octave, and it is thus desired to choose S_g and S_L so that the product of terms B and C yields an opposite gain–frequency slope. It is usual to design matching networks so that one gives "flat" response with the other being sloped. In the remainder of this communication it is assumed that $S_L=S_{22}^*$ where * implies complex conjugate (similar treatment is possible for the case where $S_g=S_{11}^*$). It should be noted that S_{22} is not in general equal to ρ_2 (reflection coefficient looking into port 2 of the device) when $S_{12}\neq 0$. However in design techniques using the unilateral approximation it is common to design an output matching network which will give $S_L=S_{22}^*$ in the band of interest.

B. Circles of Constant Gain

With $S_L=S_{22}^*$ one may derive solutions of S_g for constant "idealized" gain G_u[3] and "true" gain G_T. Following [3] and defining

$$\alpha=\frac{G_u}{G_{u\max}} \quad (4)$$

such solutions for constant G_u are given by circles in the S_g plane of center

$$\frac{\alpha S_{11}^*}{1-(1-\alpha)|S_{11}|^2}=K_1e^{-j\theta_1} \quad (5a)$$

and radius

$$\frac{\sqrt{1-\alpha}\,(1-|S_{11}|^2)}{1-(1-\alpha)|S_{11}|^2}=K_2. \quad (5b)$$

In the case of the real device the center may be calculated as

$$\frac{(S_{22}^*\triangle-S_{11})^*(1-|S_{22}|^2)}{|S_{21}|^2(1-|S_{22}|^2)/G_T+|S_{22}^*\triangle-S_{11}|^2} \quad (6a)$$

and the radius as

$$\frac{\left(\left(|S_{21}|^2(1-|S_{22}|^2)/G_T\right)^2-|S_{21}|^2\left(1-|S_{22}|^2\right)^3/G_T+|S_{22}^*\triangle-S_{11}|^2|S_{21}|^2(1-|S_{22}|^2)/G_T\right)^{1/2}}{|S_{21}|^2(1-|S_{22}|^2)/G_T+|S_{22}^*\triangle-S_{11}|^2}. \quad (6b)$$

C. Circles of Constant Error

From the relations (1) and (2) one may see that G_T and G_u are related by

$$G_T=\frac{G_u}{|1-x|^2} \quad (7)$$

with

$$x=\frac{S_gS_LS_{12}S_{21}}{(1-S_gS_{11})(1-S_LS_{22})} \quad (8a)$$

$$S_L=S_{22}^* \qquad x=\frac{S_gS_{22}^*S_{12}S_{21}}{(1-S_gS_{11})(1-|S_{22}|^2)}. \quad (8b)$$

The concept of circles of constant error may conveniently be introduced. These may be drawn in the S_g plane and are used in conjunction with circles of "ideal" and "true" gain. Setting $F=|1-x|$, calculation of solution values of S_g results in circles in the S_g plane of center

$$\frac{D_1-F^2S_{11}^*}{D_1D_1^*-F^2S_{11}S_{11}^*} \quad (9a)$$

and radius

$$\frac{F|W|}{D_1D_1^*-F^2S_{11}S_{11}^*} \quad (9b)$$

with

$$D_1=S_{11}+W, \qquad W=\frac{S_{22}^*S_{12}S_{21}}{1-|S_{22}|^2}. \quad (10)$$

III. Derivation of Condition for Zero Error Case to be Possible

It is of value to determine conditions for the existence of points in the S_g plane where the circles for a given value of G_u and G_T intersect. From (7) for zero error

$$|1-x|=1 \quad (11)$$

implying that

$$S_gS_g^*\{|S_{22}S_{12}S_{21}|^2+(1-|S_{22}|^2)\cdot(S_{11}S_{22}S_{12}^*S_{21}^*+S_{11}^*S_{22}^*S_{12}S_{21})\}-(1-|S_{22}|^2)(S_g^*S_{22}S_{12}^*S_{21}^*+S_gS_{22}^*S_{12}S_{21})=0. \quad (12)$$

One may combine this condition with the requirement that S_g lie on the circle with center and radius given by (5), and one may thus write the condition for zero error being possible as

$$E_f\geqslant 0 \quad (13)$$

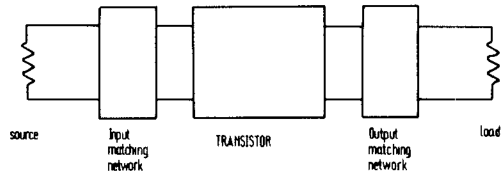

Fig. 1. Amplifier configuration under consideration.

with

$$E_f = 4(1-\alpha)\left((1-|S_{11}|^2)(1-|S_{22}|^2)\right)^2 - \{2(1-|S_{11}|^2)(1-|S_{22}|^2) \cdot |S_{11}| \cos(\phi+\theta_1) + |S_{12}S_{21}S_{22}|(1-\alpha-|S_{11}|^2)\}^2$$

where α is as in (5)

$$S_{11} = |S_{11}|e^{j\theta_1} \qquad S_{22}^*S_{12}S_{21} = |S_{22}S_{12}S_{21}|e^{j\phi}. \tag{14}$$

It is seen that $E_f \geqslant 0$ for $\alpha = 1-|S_{11}|^2$ and $E_f \leqslant 0$ for $\alpha = 1$. This implies that except in the trivial case where $S_{12}=0$ in the true device, zero error may be attained where $\alpha = 1-|S_{11}|^2$, but not where $\alpha = 1$.

IV. Derivation of Bounds on Error

The solution for bounds on error may also be obtained analytically. In doing so, the error may be expressed in terms of θ, with

$$S_g = K_1 e^{-j\theta_1} + K_2 e^{j\theta} \tag{15}$$

and x may thus be written in the form

$$x = \frac{T+Ue^{j\theta}}{V+Xe^{j\theta}} \tag{16}$$

where

$$T = K_1 e^{-j\theta_1} \frac{S_{22}^*S_{12}S_{21}}{1-|S_{22}|^2}, \qquad U = K_2 W, \qquad V = 1 - S_{11}K_1 e^{-j\theta_1},$$

$$X = -S_{11}K_2.$$

Hence $F^2 = |1-x|^2$ may be evaluated, and setting $\partial(F^2)/\partial\theta = 0$ implies

$$GD \sin(\theta+\theta_D) - HJ \sin(\theta+\theta_H) - HD \sin(\theta_D - \theta_H) = 0 \tag{17}$$

where $G = TT^* + UU^* - 2\mathrm{Re}(TV^* + UX^*)$, $J = VV^* + XX^*$, $D = 2|V^*X|$, $H = 2|T^*U - T^*X - UV^*|$, $\theta_H = \angle(T^*U - T^*X - UV^*)$, and $\theta_D = \angle V^*X$ from which θ is obtained, with the maximum and minimum values of true gain got from (1). Also for $F=1$ (zero error)

$$G + H\cos(\theta+\theta_H) = 0 \tag{18}$$

with

$$\theta = \cos^{-1}\left(-\frac{G}{H}\right) - \theta_H \tag{19}$$

which yields an alternative condition for the zero error case, namely that

$$\left|\frac{G}{H}\right| \leqslant 1. \tag{20}$$

These expressions may be incorporated quite readily in a computer routine which permits evaluation of maximum possible error when designing for a given G_u value, in terms of this quantity and of the S-parameters of the device, where $S_L = S_{22}^*$.

V. Results Obtained with Microwave Devices

Calculations using the relations derived in earlier sections have been undertaken. Cases treated have been 1) Hewlett-Packard 1-μm gate FET device [2] at 10 GHz, the 1-μm gate packaged FET [1] at 2) 4 GHz, and 3) 8 GHz, and 4) the Plessey COD device [4] at 8 GHz. S-parameter information, derived stability quantities K and B_1 (see [3]) and maximum gain figures are given in Figs. 2 and 3. It is seen that B_1 is positive in all cases, with $K>1$ in cases 1, 2, and 3, but with $K<1$ in case 4. This indicates that in case 4 the device is potentially unstable for some combination of source and load terminations, and that cases 1, 2, and 3 are unconditionally stable at the frequency of interest. Some consequences of this instability will be seen. Calculations were undertaken for α values from $\alpha=0.05$ to $\alpha=0.95$ in 0.10 increments. Values of the quantity E_f (14) and of upper and lower bounds on gain achievable with the "true" device are shown in Figs. 4(a)–7(a). These upper and lower bounds are, respectively, the designated G_{T1} and G_{T2} on the diagrams. Also shown are circles of constant G_u in the S_g plane and indicated on these are S_g points for zero error (where such exist) and also S_g points corresponding to maximum and minimum "true" gain figures.

In cases 1 and 2 it is noted that considerable error may result from the idealization, but less error may result in case 3. In case 4 effects of potential instability may be noted. The stability circle [3] is drawn in the S_g plane, with points on this circle corresponding to infinite G_T. It is seen that an upper bound is thus undefined for S_g solutions intersecting the stability circle. In Fig. 6 (c) the effects of near coincidence of the "true" and "unilateral" circles may be seen. As expected therefore, it is seen that use of a design technique based on unilateral idealization may be of little value in the case of a potentially unstable device. It may be desirable in such a case to choose suitable values of S_g far from the stability circle and perhaps use an impedance-approximation technique such as in [5].

VI. Conclusions

Relations presented permit calculation of bounds on error which may arise when using a design technique where S_{12} is set to zero with other S-parameters unchanged. Results have been presented for some typical devices and it is seen that considerable error may arise

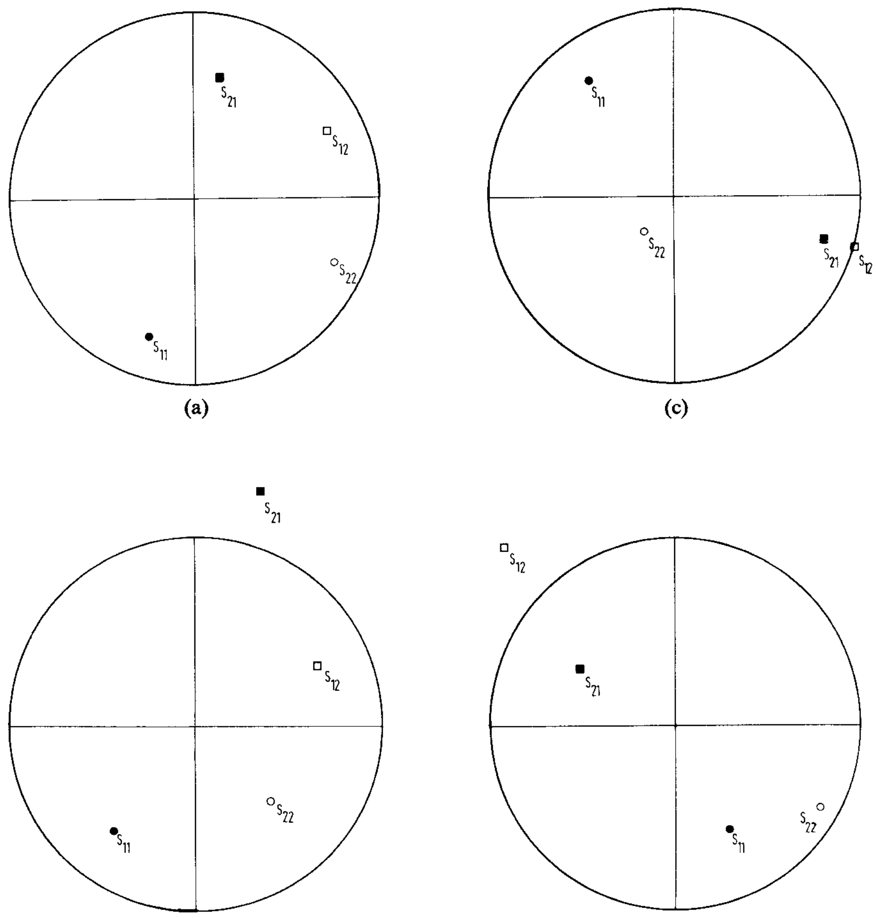

Fig. 2. S-parameter data for microwave transistors. (Circle corresponds to 1.0 for S_{11}, S_{22}, to 0.05 for S_{12}, and 2.0 for S_{21}). (a) For HP 1-μm chip device, at 10 GHz ($S_{11}=0.771\ \angle-108.9$, $S_{12}=0.040\ \angle 26.6$, $S_{21}=1.34\ \angle 78.3$, $S_{22}=0.826\ \angle-25.5$). (b) For HP 1-$\mu$m packaged device, at 4 GHz ($S_{11}=0.712\ \angle-129.0$, $S_{12}=0.036\ \angle 25.9$, $S_{21}=2.59\ \angle 74.7$, $S_{22}=0.563\ \angle-42.4$). (c) For HP 1-$\mu$m packaged device, at 8 GHz ($S_{11}=0.674\ \angle 126.7$, $S_{12}=0.050\ \angle-16.8$, $S_{21}=1.65\ \angle-16.1$, $S_{22}=0.244\ \angle-132.0$). (d) For Plessey device at 8 GHz ($S_{11}=0.620\ \angle-64.0$, $S_{12}=0.066\ \angle 134.0$, $S_{21}=1.19\ \angle 150.0$, $S_{22}=0.880\ \angle-30.0$).

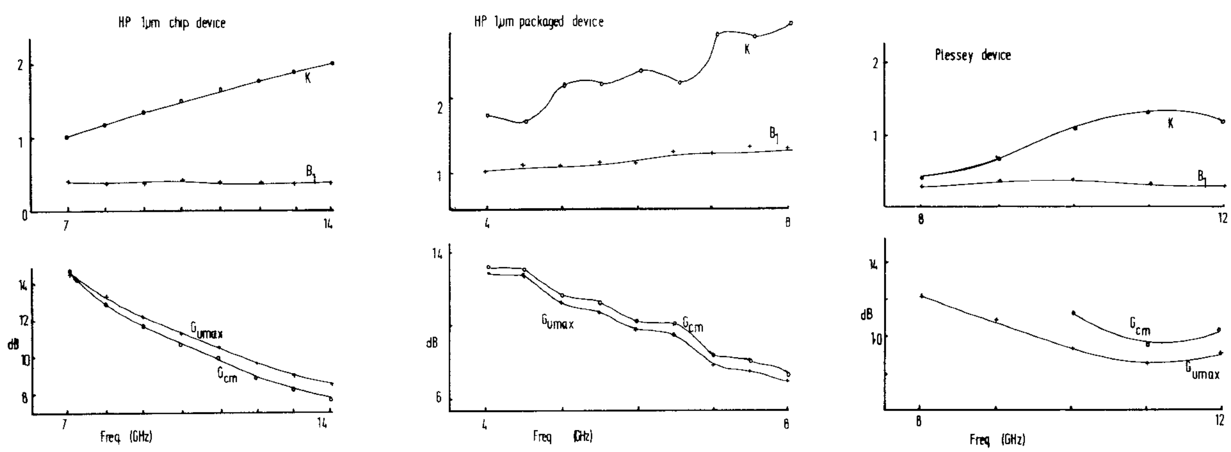

Fig. 3. Stability quantities (K and B_1) and gain quantities (G_{cm} and G_{umax}) for devices.

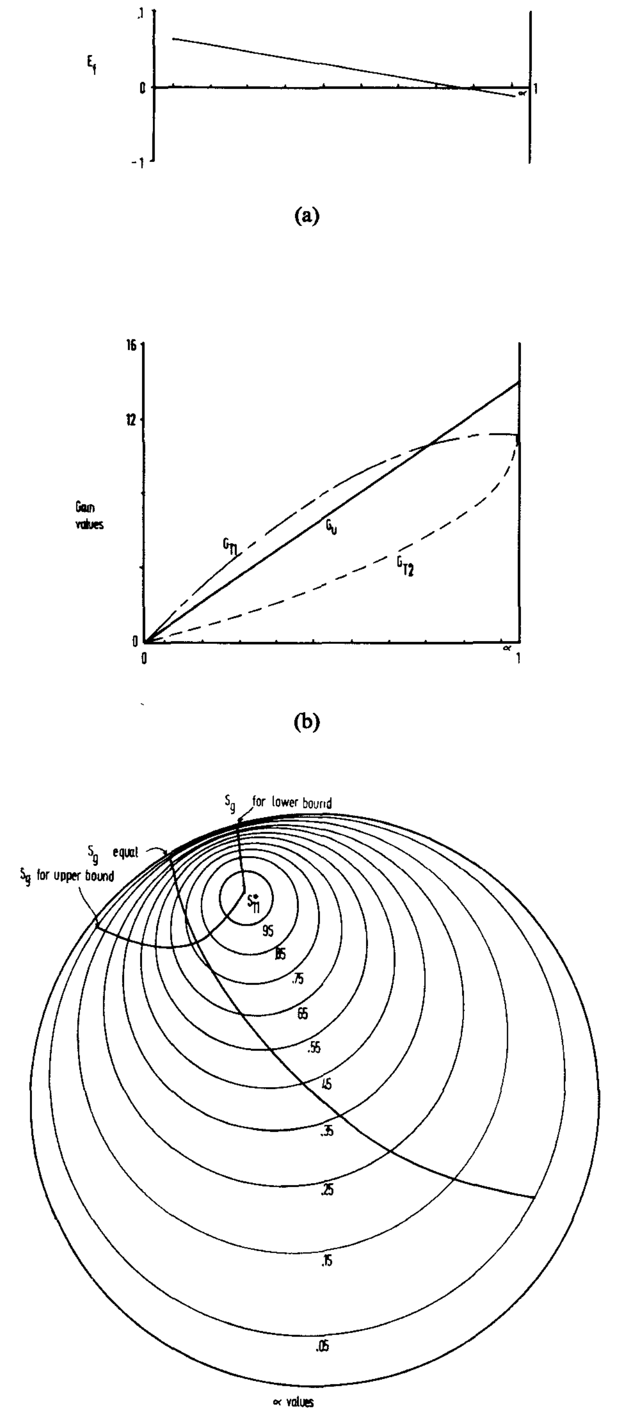

Fig. 4. (a) Plot of E_f as a function of α (case 1). (b) Plot of G_u, G_{T1}, and G_{T2} as functions of α. Zero crossings of E_f at α values of -30.53 and 0.794. (c) S_g plane with circles of constant α, showing solutions for upper and lower bounds and also for zero-error case.

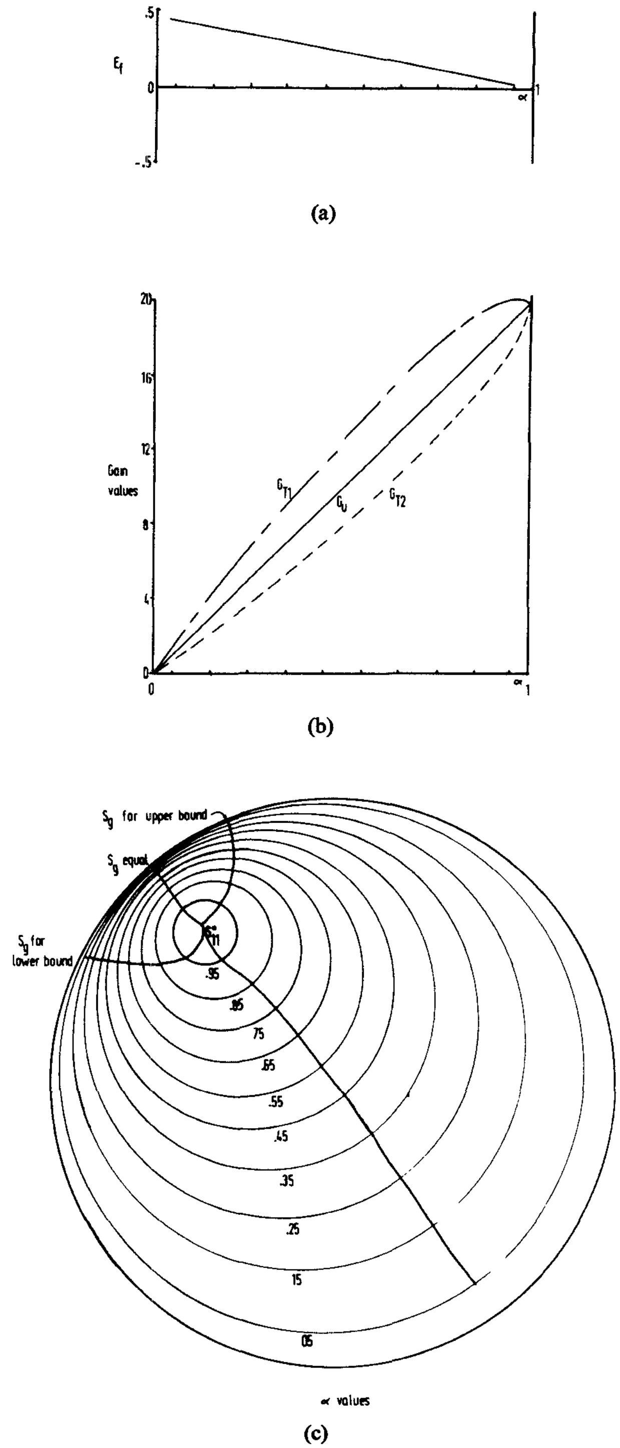

Fig. 5. (a) Plot of E_f as a function of α (case 2). (b) Plot of G_u, G_{T1}, and G_{T2} as functions of α. Zero crossings of E_f at α values of -164.13 and 0.99979. (c) S_g plane with circles of constant α, showing solutions for upper and lower bounds and also for zero-error case.

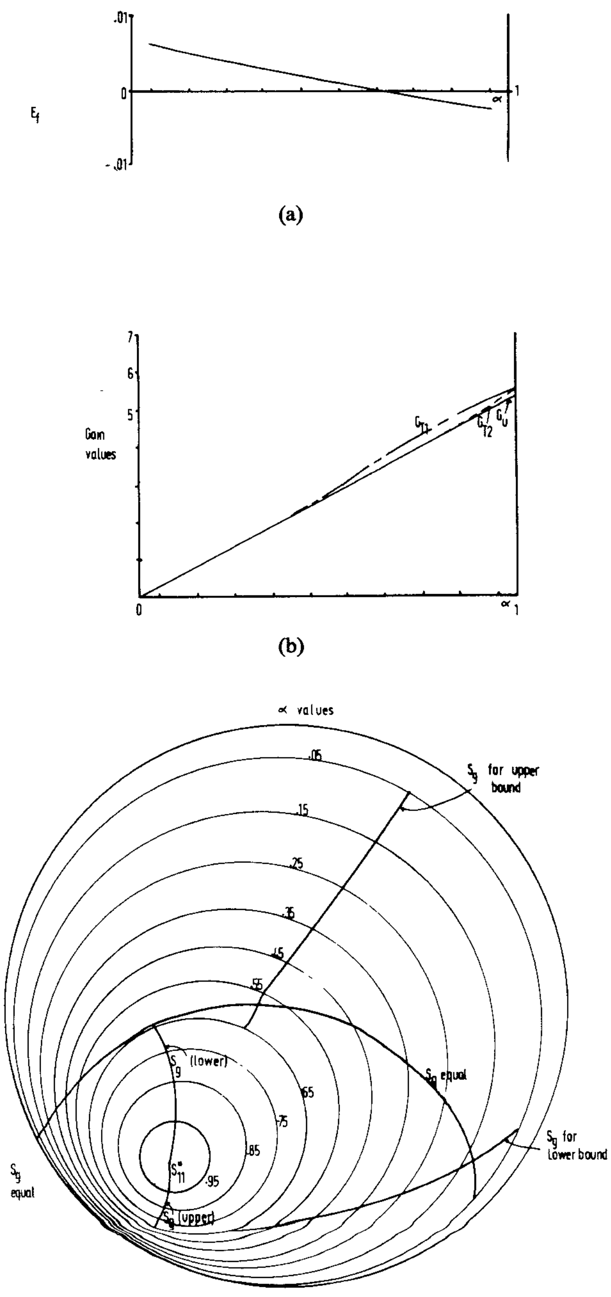

Fig. 6. (a) Plot of E_f as a function of α (case 3). (b) Plot of G_u, G_{T1}, and G_{T2} as functions of α. Zero crossings of E_f at α values of -2541.9 and 0.646. (c) S_g plane with circles of constant α, showing solutions for upper and lower bounds and also for zero-error case.

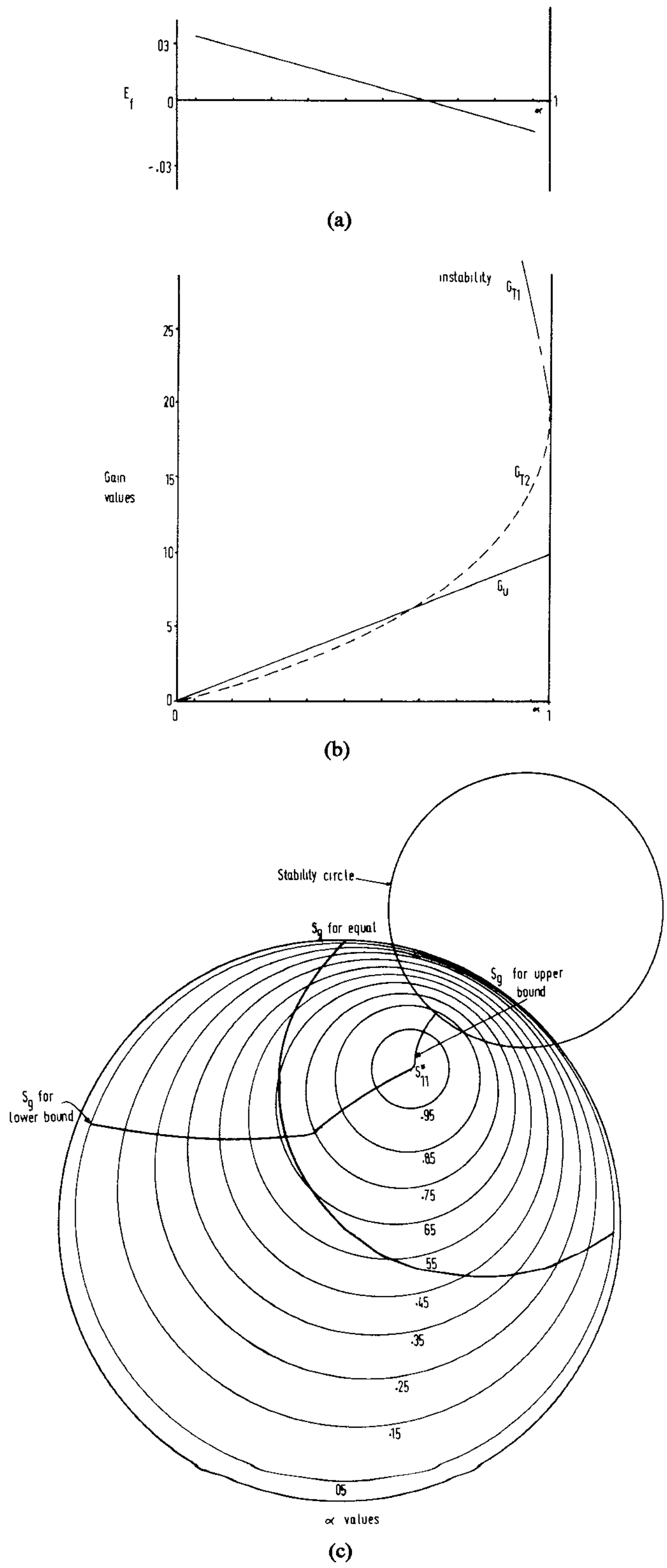

Fig. 7. (a) Plot of E_f as a function of α (case 4). (b) Plot of G_u, G_{T1}, and G_{T2} as functions of α. Zero crossings of E_f at α values of -10.85 and 0.667. (c) S_g plane with circles of constant α, showing solutions for upper and lower bounds and also for zero error case.

from this cause particularly in the case of a potentially unstable device.

Conditions have also been obtained for the existence of points in the S_g plane for a given G_u corresponding to equality of gain figures for the "true" and "ideal" devices. The contour techniques and expressions given in this paper permit ready evaluation of the possible errors introduced by using some of the common "unilateral" design methods. With the assumption that $S_L = S_{22}^*$ it is possible to calculate such error bounds in terms of the transistor S-parameters and of the required gain. Use of these techniques thus provides an additional insight into the suitability of a transistor for a given application.

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Electrical Characteristics of Metal-Semiconductor Junctions

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***Abstract*—The current–voltage characteristic of metal–semiconductor junctions is described by a simple equation which is the product of an exponential and a hyperbolic sine function if one includes the effect of tunneling. In the case of equal current flow via tunneling and via thermionic emission the conductance becomes a hyperbolic cosine function of the applied voltage. Devices displaying such characteristics appear attractive for use in harmonic frequency converters.**

I. Introduction

THE CARRIER transport mechanisms which determine the electrical properties of metal–semiconductor junctions have been investigated theoretically and experimentally since the early work of Braun [1]. Various types of junctions were named after the major investigators who made new contributions toward the understanding of the rectification process, typical examples being the Schottky diode [2] and the Mott diode [3]. A common characteristic of both these junctions is that the current is an exponential function of the applied voltage with the easy direction of electron flow being from the semiconductor to the metal.

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The purpose of this paper is to present equations for the current, the conductance, and the shot noise of metal–semiconductor junctions that encompass both a forward and a reversed direction of easy current flow. A reverse direction of rectification was already forseen by Wilson in 1932 [4] who postulated that the major mechanism of current transport was quantum-mechanical tunneling of electrons from the metal into the semiconductor through a thin interfacial barrier. It was not possible to fabricate such junctions for several decades. However, recent advances by Cho [5] in growing extremely thin MBE layers make it possible to achieve a controlled ratio of the current flow via tunneling and via thermionic emission. Junctions of this type appear useful for the development of hybrid integrated frequency converters because the required pump frequency can be a subharmonic of that which is used in conventional mixers and upconverters.

II. Current–Voltage Characteristic

The analysis of scattering and transport mechanisms of the carriers in a metal–semiconductor junction is rather complicated as shown in a recent paper by Salardi *et al.* [6]. There are, however, some simplifications possible

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